NATIONAL ADVISORY COMMITTEE
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TECHNICAL NOTE 2441

OPTICAL METHODS INVOLVING LIGHT SCATTERING FOR MEASURING

SIZE AND CONCENTRATION OF CONDENSATION PARTICLES

IN SUPERCOOLED HYPERSONIC FLOW

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SUMMARY

Optical methods involving light scattering for measuring the size and concentration of condensation particles in supercooled hypersonic air flow are discussed. Two methods based on scattered-light measurements and on transmitted-light measurements are given which can be used for obtaining quantitative measurements provided (1) that steady-state conditions can be achieved during the time required for the measurement of light intensity, (2) that the condensation particles are approximately spherical in shape and essentially uniform in size, and (3) that the index of refraction of the condensation particles is known approximately.

For verification of these methods the radii of ammonium-chloride particles were determined, and a comparison of these results with measurements made by a method independent of light-scattering theory and with measurements made by Langstroth, Gillespie, and Pearce gave good agreement.

With the use of the light-scattering methods, the size and concentration of condensation particles in supercooled hypersonic air flow were measured.

INTRODUCTION

The problem of condensation of the components of air in hypersonic flow has arisen in connection with work in the Langley 11-inch hypersonic tunnel and has been reviewed by Becker in reference 1. Because of the rapid expansion and cooling of the air in the nozzle of this tunnel, the temperature of the air rapidly drops below that at which it liquefies. Under these conditions, fog has been observed in the test section. This appearance of fog is accompanied by changes in

the aerodynamic parameters measured in the tunnel. If the air is pre-
heated sufficiently to keep the temperature in the test section above
the condensation point, fog can be prevented from forming. However,
because of the very high expansion ratio, large quantities of heat must
be transferred during the test run at a fairly rapid rate if the appear-
ance of fog is to be prevented; therefore, high-powered heaters which
are very expensive to install and operate are required if fog-free
hypersonic flow is to be obtained.

In order to obtain a more satisfactory solution to the problem,
a satisfactory theory by which the presence of condensation in such a
tunnel can be predicted is needed. The phenomenon of condensation in
hypersonic flow is, in fact, the subject of much controversy. Several
plausible explanations of the appearance of the fog have been advanced
and are summarized in reference 1. However, detailed quantitative
information concerning the nature of the condensation particles is
necessary before a satisfactory understanding of the phenomenon can be
had.

The purpose of this paper, then, is to provide experimental methods
for measuring the size and concentration of condensation particles in
supercooled hypersonic flow in order to help resolve some of the ques-
tions concerning the nature of this condensation.

A fundamental requirement of any proposed method of measuring con-
densation particles is that the air flow in which the fog appears be
undisturbed by the measuring process. This requirement suggests opti-
cal methods of measurement. Optical methods involving light scattering
were investigated because the particles were observed by their ability
to scatter light. An effective presentation of light scattering by
condensation particles is shown in figures 1 and 2.

The author is indebted to the Computation Laboratory of the National
Applied Mathematics Laboratories of the National Bureau of Standards for
extending the tables of angular-light-intensity-distribution functions
needed for this project.

<div align="center">SYMBOLS</div>

I intensity of beam of light after it passes through a length l
           of condensation particles

I_θ intensity of light scattered at any angle θ

I_0 incident intensity of beam of light

θ angle between direction of incident beam and reverse direction of observed scattered light

λ wavelength of light source

R distance from particle to point of observation

$i_1(\theta), i_2(\theta)$ functions of angular distribution of intensity proportional to intensities of plane-polarized components of scattered light; precisely defined in first section of appendix

K propagation constant of electromagnetic wave in condensation particle

r particle radius

$\alpha = Kr = \dfrac{2\pi r}{\lambda}$

n particles per unit volume

l distance light travels through condensation particles

k scattering-area coefficient

THEORY OF LIGHT SCATTERING

Intensity of Scattered Light

The angular distribution of intensity and degree of polarization of the light scattered by a fog is related to both the size and index of refraction of the fog particles. This relationship permits the calculation of fog-particle sizes from observations of the distribution of intensity and polarization of the scattered light.

Maxwell's equations describing the behavior of electromagnetic waves have been used to derive a theory of light scattering by small particles. If an electromagnetic wave impinges on a body, a forced oscillation of all free and bound charges within the body occurs. This oscillation occurs at the same frequency as that of the applied field. These constrained moving charges in turn set up secondary fields. The net field, then, at any point is made up of the vector sum of both the primary incident field and the secondary fields. The solution of Maxwell's wave equations in spherical coordinates for the condition of an electromagnetic wave incident on a sphere was derived by

Gustav Mie in 1908. The complete derivation is given in a compact
form by Stratton (reference 2).

The general solution for particles of any size illuminated by a
monochromatic light beam of unit intensity can be written as

$$I_\theta = \frac{\lambda^2}{8\pi^2 R^2}\left[i_1(\theta) + i_2(\theta)\right]$$

where

θ angle between incident beam and direction of observation

I_θ intensity of light scattered at any angle θ

λ wavelength of the monochromatic light source

R distance from the particle to the point of observation

$i_1(\theta), i_2(\theta)$ functions of angular distribution of light intensity;
                precisely defined in the first section of the appendix

The angular-intensity-distribution functions $i_1(\theta)$ and $i_2(\theta)$
are proportional to the intensities of the two plane-polarized com-
ponents scattered by a particle illuminated with monochromatic light
(see fig. 3). The function $i_1(\theta)$ is the component of the scattered
light the electric vector of which is perpendicular to the plane of
observation (the plane containing the incident ray and the point of
observation), and $i_2(\theta)$ is the component the electric vector of
which is in the plane of observation. When the particle is illuminated
by a light of unit intensity, the components of the intensity of the
light scattered per unit solid angle as measured at an angle θ from
the incident ray are given by

$$\frac{\lambda^2 i_1(\theta)}{8\pi^2}$$

and

$$\frac{\lambda^2 i_2(\theta)}{8\pi^2}$$

The scattering theory, as given here, is derived for a single particle. For the results to be applicable to measurements of fog particles, the light must be scattered only once in going from the incident beam to the observer. This type of scattering can be assumed to occur, when the particles are separated by a distance of 100 times the particle radius. Scattering theory further assumes that the effect of each particle can be added to determine the net effect of a group of particles.

Determination of Particle Size by Light-Scattering Method

A study of the equations of Mie's solution in the first section of the appendix indicates the complexity and difficulty in attaining numerical solutions. Fortunately, however, the application of these equations to the determination of particle size can be made less difficult by considering three limiting cases (reference 2) which serve to reduce Mie's solution to a more readily usable form.

The three limiting cases are defined in terms of α which equals Kr where K is the propagation constant of an electromagnetic wave in the spherical condensation particle and r is the particle radius. Or they can be defined in terms of the relation of r to λ, the wavelength of the light, since

$$K = \omega\sqrt{\mu\epsilon}$$

$$v = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\omega = 2\pi f = \frac{2\pi v}{\lambda}$$

and therefore

$$K = \frac{\omega}{v} = \frac{2\pi}{\lambda} \approx \frac{1}{\lambda}$$

where μ is the permeability of the particle, ϵ is its permittivity, v is the velocity of the wave in the particle, ω is the angular frequency of the wave, and f is its frequency.

(1) For the first limiting case, assume $\alpha \ll 1$. This case is then the case of particles of radii very much less than the wavelengths of the light employed. True solutions and the fine colloidal solutions have particles with sizes falling in this group. In this

case, the infinite series representing $i_1(\theta)$ and $i_2(\theta)$ converge very rapidly, and only the first-order electric oscillation term need be considered. This mode of oscillation corresponds to that of an electric dipole. This particular solution to Mie's equations is the well-known Rayleigh law of scattering (references 3 and 4):

$$I_\theta = \frac{9\pi^2}{2R^2}\left(\frac{m^2 - 1}{m^2 + 2}\right)^2 \frac{V^2}{\lambda^4}(1 + \cos^2\theta)$$

where m is the index of refraction of the particle and V is the particle volume.

(2) For the second case, assume $\alpha \gg 1$. Now the particle radius is very much greater than the wavelength of the light employed. Coarse suspensions and most natural fog water droplets have particle sizes in this group. In this case, Mie's solution reduces to the much more simple Huygen's principle as used in diffraction and reflection problems in physical optics (reference 5). With increasing particle size, the practical value of Mie's solution diminishes, and an approximate macroscopic theory should be used. This theory is based on simple reflection and refraction of a wave at the boundary between two surfaces. The intensity of the light reflected is no longer dependent on the droplet size but rather is a function of index of refraction and angle of incidence.

(3) For the third case, assume $\alpha \approx 1$, where the particle radii are of the same order of size as the wavelength of the light employed. A major portion of all colloidal solutions and fine suspensions have particle sizes in this group. In this case, no simplifying reductions occur; however, the Computation Laboratory of the National Applied Mathematics Laboratories of the National Bureau of Standards has computed and tabulated the series of Mie's solution for particles of sizes ranging from $\alpha = 0.5$ to $\alpha = 6.0$ and for particles with index of refraction m from 1.33 to 2.00. (See reference 6.)

Each of the three groups in the preceding section has distinctive light-scattering characteristics due to which particles in an aerosol or fog can be classified according to size by simple light-scattering measurements.

If a fog of unknown particle size is illuminated by a beam of monochromatic light and the intensity of the scattered light is measured simultaneously at two angles θ_1 and θ_2, which are symmetrical about a line drawn normal to the direction of incidence, then classification of the particles in the fog according to size is possible. As an example,

if the angles chosen for the scattering measurement are 20° and 160°, as measured from the direction of incidence, then the following three conditions can occur:

(1) If the intensities at 20° and 160° are equal, then the particle falls in the first group and the radius is less than approximately 0.02 of the wavelength of the light source. This upper limit for the Rayleigh type of scattering can be determined from the values computed from Mie's solution.

(2) If the intensity of scattered light at 20° is negligible compared with the intensity at 160°, then the particle falls in the second group, and its radius is greater than approximately 2 wavelengths of the light source.

(3) If the intensities at 20° and 160° are not equal and that at 20° is not negligible compared with that at 160°, then the particle radius is in the third group and its size can be determined more exactly. Within this group, the ratio $i_1(\theta)/i_2(\theta)$ or the polarization of the scattered light at a known angle of scattering provides a measure of particle size when used with the previously mentioned tabulation of Mie's solution. Unless absolute intensities of $i_1(\theta)$ and $i_2(\theta)$ are known however, this method does not provide a unique solution to the particle-size problem for all sizes within this group since there is more than one particle size which will yield the same ratio $i_1(\theta)/i_2(\theta)$. A unique solution can be obtained, however, when the particle size is in the third group, without the need for measuring absolute intensities if the ratio $i_2(\theta_1)/i_2(\theta_2)$ is used. Since this measurement of $i_2(\theta_1)/i_2(\theta_2)$ can be used to establish the group to which the particles belong, the measurement of a single ratio of intensities can be used to establish both the size group and particle size if the particles are in the third group.

Although Mie's solution was derived for spherical particles, small deviations from the spherical shape produce no large error in the calculation. In choosing angles θ_1 and θ_2 it should be noted that measurements of scattered light at angles close to the incident beam are desirable. The relative intensity $i(\theta_1)/i(\theta_2)$ is a function of particle size. If θ_1 and θ_2 are chosen to be close to the line of incidence, the ratio $i(\theta_1)/i(\theta_2)$ is found to have greater variation for small changes in particle size.

Determination of Uniformity of Particle Size

Since the preceding method requires a reasonably uniform particle
size in the fog under investigation, a convenient method of determining
particle-size uniformity is necessary..

When a fog is illuminated with white light, the angular distribution
of scattered light is different for each color component of the white
light. If the particles in a fog are uniform in size, one wavelength
is scattered with intensity greater than all others at each angle of
scatter. Thus if the angle of observation of $i_1(\theta)$ is varied from

0° toward 180°, a sequence of color spectra in the order violet, blue,
green, yellow, orange, and red is seen if the particles are uniform in
size. Study of the tabulated solution of Mie's equation (reference 6)
reveals that the number of spectra is roughly equal to 10^5 times the
particle radius in centimeters. These spectral sequences are known as
higher-order Tyndall spectra.

The purity and brightness of the spectra increase with uniformity
of particle size. For smaller-size particles the number of spectra is
reduced until at a radius of approximately 10^{-5} centimeters the spectral
sequences disappear. Thus, higher-order Tyndall spectra cannot be used
in determining particle-size uniformity for particles of radius less
than 10^{-5} centimeters. For particles having a radius less than this
value the angular distribution of scattered light is almost independent
of wavelength. For small particles the effective scattering area
increases for decreasing values of wavelength. Thus, if small particles
are illuminated with white light, the scattered light will appear
bluish white and the transmitted light will appear reddish. This differ-
ence is illustrated by the blue color of the scattered light from the
sky, and the red color of the transmitted light of the sun as it sets.

Intensity of Transmitted Light

In the foregoing sections, the determination of particle size from
measurements of the characteristics of the scattered light was presented.
An alternate approach to the problem utilizes the change in the charac-
teristics of the transmitted light due to the subtraction of the scat-
tered light. The use of light-transmission measurements is based on the
following relations:

If a beam of light of intensity I_0 impinges on a region of con-
densation particles of radius r and concentration n particles per
unit volume, the intensity of the beam I after it passes through a

length l of such particles is given by the basic transmission equation as

$$I = I_0 e^{-k\pi r^2 n l}$$

The term $k\pi r^2$ represents the effective area for light scattering for each particle.

The scattering-area coefficient k is a function of particle size, wavelength, and index of refraction of the particle and is defined mathematically in the second section of the appendix. The expression

$$I_0 \left(1 - e^{-k\pi r^2 n l}\right)$$

represents the sum of all the light scattered over the distance l. Since this sum is a function of k and for a given group of fog particles of uniform size and uniform concentration k is a function of wavelength λ only, then the transmission I/I_0 is a function of wavelength only.

Determination of Particle Size and Concentration

by Transmission Method

If k_1 is the scattering-area coefficient for the particles in a given fog at a wavelength λ_1 and k_2 is the scattering-area coefficient for these same particles at a wavelength λ_2, then the intensities of the light for the two conditions are

$$I_1 = I_{0_1} e^{-k_1 \pi r^2 n l}$$

$$I_2 = I_{0_2} e^{-k_2 \pi r^2 n l}$$

where I_{0_1} is the incident light intensity at λ_1 and I_{0_2} is the incident light intensity at λ_2. These equations can be written as

$$\log_e \frac{I_{0_1}}{I_1} = k_1 \pi r^2 n l$$

$$\log_e \frac{I_{0_2}}{I_1} = k_2 \pi r^2 n l$$

Now if these two equations are divided,

$$\frac{k_1}{k_2} = \frac{\log_e I_{0_1} - \log_e I_1}{\log_e I_{0_2} - \log_e I_2}$$

Therefore, if measurements are made of the transmission at two wave-lengths λ_1 and λ_2, the ratio k_1/k_2 can be determined.

Since

$$\alpha = \frac{2\pi r}{\lambda}$$

define

$$\alpha_1 = \frac{2\pi r}{\lambda_1}$$

$$\alpha_2 = \frac{2\pi r}{\lambda_2}$$

then

$$\frac{\alpha_1}{\alpha_2} = \frac{\lambda_2}{\lambda_1}$$

From reference 6 or from table 1 of the present paper, a curve of k as a function of α can be obtained for the index of refraction of the particles being studied. If this curve is plotted with logarithmic scales, a simple geometric process can be used to determine the particle size. For illustrative purposes, a curve of k as a function of α for an index of refraction of 1.44 is plotted in figure 4.

The ratio k_1/k_2, just determined, represents a fixed distance along the ordinate of figure 4. Similarly, the ratio $\frac{\alpha_1}{\alpha_2} = \frac{\lambda_2}{\lambda_1}$ repre-sents a fixed distance along the abscissa of figure 4. If now, the curve is examined for two points, separated by an abscissa difference

of λ_2/λ_1 and an ordinate difference of k_1/k_2, the values of α corresponding to these two points can be read off the abscissa scale. The radius can be found from

$$r = \frac{\alpha_1 \lambda_1}{2\pi}$$

or

$$r = \frac{\alpha_2 \lambda_2}{2\pi}$$

The chief advantages of this method are that it permits a study to be made of the growth of particles, and, in addition, a single set of measurements suffices for both concentration and size.

Reference 6 contains a tabulation of k as a function of α for indices of refraction from 1.33 to 2.00. The Computation Laboratory of the National Bureau of Standards has extended the computations of reference 6 for this project. Included in this extension is a tabulation of k as a function of α for an index of refraction of 1.20 which approximates the value of the index for both oxygen and nitrogen. In addition, the angular-light-intensity-distribution functions $i_1(\theta)$ and $i_2(\theta)$ have been calculated for this index of refraction. These extensions are given in table 2.

The concentration of particles can be easily determined when the particle radius is known. Solving the basic transmission equation for n gives

$$n = \frac{\log_e I_0 - \log_e I}{k\pi r^2 l}$$

If the particle radius has been determined by measurements of the scattered light, a simple measurement of light transmission at one wavelength will be necessary for the calculation of the particle concentration; if the radius has been determined by the light-transmission method, no further measurements need be made to determine concentration.

EXPERIMENTAL VERIFICATION OF LIGHT-SCATTERING METHODS

In order to determine the validity of the light-scattering methods, experiments were conducted to measure the size of particles in a

chemical fog. An aerosol of ammonium-chloride particles was prepared by the combination of the vapors of hydrochloric acid and ammonium hydroxide. The particles were found by preliminary measurements to fall in the third group ($r \approx \lambda$). Measurements of particle size were made by three different methods, and the results were compared. In addition, experimental results of reference 7 are given.

For the light-scattering measurements, a monochromatic light consisting of the green line of the mercury spectrum with $\lambda = 5461$ angstrom units was used to illuminate the ammonium-chloride aerosol. (See fig. 5.) An average particle radius of 4.1×10^{-5} centimeters was determined; however, a large variation existed in the particle-size measurements. Radii as small as 2.5×10^{-5} centimeters and as large as 5.0×10^{-5} centimeters were measured in succeeding experiments. This variation could be only partially attributed to the inherent difficulties in measuring scattered light.

The particles formed in the combination of the vapors are electrically charged. Because of this charge the rate of aggregation tends to be reduced and thus a more nearly constant particle size will be produced. However, a small but definite growth of particles does occur during the experiment; therefore, making two successive identical measurements becomes very difficult.

For the light-transmission measurements, a fog of ammonium-chloride particles was illuminated by the green line (5461 A.) and the blue line (4358 A.) of the mercury spectrum alternately. (See fig. 6.) Both size and concentration of particles were determined. Particles with radii varying between 2.9×10^{-5} centimeters and 4.2×10^{-5} centimeters were measured. This result is in good agreement with the light-scattering measurements.

Again, the particles of ammonium-chloride were found to grow during the experiment. The particle size was found to be a function of the turbulence in the chamber and the time elapsed since the formation of the particle. This observation explains the variation in sizes measured by these methods.

A measurement of particle size which was completely independent of light-scattering theory was necessary to provide a reference with which results obtained by using light-scattering methods could be compared. One such method which makes use of Stokes' law is as follows:

If a small spherical particle of radius r passes through a gas of viscosity η at a constant velocity v, the force F which is opposing the motion of the particle is given by Stokes' law as

$$F = 6\pi r \eta v$$

If the particle is falling because of the force of gravity, the force tending to accelerate the particle is given by

$$F = mg = \frac{4}{3}\pi r^3(\rho - \rho')g$$

where ρ is the density of the particle and ρ' is the density of the medium. When the particle is falling at a constant velocity,

$$\frac{4}{3}\pi r^3(\rho - \rho')g = 6\pi r \eta v$$

$$r = \sqrt{\frac{9\eta v}{2(\rho - \rho')g}}$$

Thus, by measuring the constant velocity at which the particles in the fog fall, determination of the particle radius is possible.

An experiment was performed by applying Stokes' law for the measurement of particle size in an ammonium-chloride fog. The average velocity of particle fall was found to be 0.00635 centimeter per second. With the use of the following parameters,

$$\eta = 179.6 \times 10^{-6} \text{ grams per second-centimeter}$$

$$\rho = 1.527 \text{ grams per centimeter}^3$$

$$\rho' \approx 0.001 \text{ grams per centimeter}^3$$

$$g = 979.6 \text{ centimeter per second}^2$$

the radius calculated was 5.85×10^{-5} centimeters.

The experimental results of reference 7 provide another means of verifying the light-scattering methods. In these experiments the average particle mass of ammonium chloride was measured and the growth of particles during aging was noted. The mass was noted to vary with air velocity and time. An average particle mass in still air of 2×10^{-13} grams was found after 2 minutes and 4×10^{-13} grams after 5 hours. Since the density of ammonium chloride is 1.527 grams per

cubic centimeter, the average particle radius, if spherical particles
are assumed, is calculated to be

$$\frac{4}{3}\pi r^3(1.527) = 2 \times 10^{-13} \text{ to } 4 \times 10^{-13}$$

$$r = 3.1 \times 10^{-5} \text{ to } 6.3 \times 10^{-5} \text{ centimeters}$$

For convenience in comparing all the foregoing experimental results,
a summary of ammonium-chloride-particle radii is given in the following
table:

	Radius (cm)
Scattered-light measurement	4.1×10^{-5}
Transmitted-light measurements	2.9×10^{-5} to 4.2×10^{-5}
Measurement utilizing Stokes' law	5.85×10^{-5}
Measurements of reference 7	3.1×10^{-5} to 6.3×10^{-5}

The agreement between the various methods is a verification of light-
scattering methods.

APPLICATION OF LIGHT-SCATTERING METHODS TO MEASUREMENT OF CONDENSATION
PARTICLES IN SUPERCOOLED HYPERSONIC AIR FLOW

The size and concentration of condensation particles in supercooled
hypersonic air flow were measured by the light-scattering method and
light-transmission method.

The experimental arrangements used are given in figures 7 and 8.
The air was expanded through a nozzle which produced a temperature ratio
of 0.096 corresponding to a pressure ratio of 0.00028 when no conden-
sation takes place. With the assumption that the condensation particles
were nitrogen or oxygen, the measurements gave a particle radius of the
order of 5.0×10^{-6} centimeters and concentrations of the order of
10^{10} particles per cubic centimeter. If the particles measured were
really droplets of water which has an index of refraction of 1.33
rather than oxygen or nitrogen with an index of 1.20, the error in the
particle-size measurement would be only 5 percent.

Since only a single scattering has been assumed to occur when the
particles are separated by a distance of 100 times the particle radius,
for particles of radius 10^{-6} centimeters concentrations up to 10^{12} parti-
cles per cubic centimeter should yield single scattering. Since the

tunnel concentrations are of the order of 10^{10} particles per cubic centimeter, single scattering can be assumed for these measurements.

CONCLUDING REMARKS

Optical methods involving light scattering for measuring the size and concentration of condensation particles in supercooled hypersonic air flow have been discussed. Two methods based on scattered-light measurements and on transmitted-light measurements have been given which can be used for obtaining quantitative measurements provided (1) that steady-state conditions can be achieved during the time required for the measurement of light intensity, (2) that condensation particles are approximately spherical in shape and essentially uniform in size, and (3) that the index of refraction of the condensation particles is known approximately.

Langley Aeronautical Laboratory
    National Advisory Committee for Aeronautics
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APPENDIX

SOLUTION OF WAVE EQUATION AND EXPRESSION

FOR SCATTERING-AREA COEFFICIENT

Mie's Solution of the Wave Equation

The solution of Maxwell's wave equation in spherical coordinates
has been derived by Gustav Mie (see reference 2) for the case of a plane
wave incident on a sphere of radius r and with an index of refraction
tion m. The angular-light-intensity-distribution functions $i_1(\theta)$
and $i_2(\theta)$ are proportional to the plane-polarized light components
scattered at an angle θ by the sphere. The planes of oscillation
of i_1 and i_2 are, respectively, perpendicular to and in the plane
of observation. (See fig. 3.) The functions $i_1(\theta)$ and $i_2(\theta)$ are
given as follows (see reference 6):

$$i_1(\theta) = \left| \sum_{n=1}^{\infty} \left\{ \frac{a_n}{n(n+1)} z_n + \frac{b_n}{n(n+1)} \left[X z_n - \left(1 - X^2\right) z_n' \right] \right\} \right|^2$$

$$i_2(\theta) = \left| \sum_{n=1}^{\infty} \left\{ \frac{a_n}{n(n+1)} \left[X z_n - \left(1 - X^2\right) z_n' \right] + \frac{b_n}{n(n+1)} z_n \right\} \right|^2$$

where

$$a_n = (-1)^{n+\frac{1}{2}} (2n+1) \frac{S_n'(\beta) S_n(\alpha) - m S_n'(\alpha) S_n(\beta)}{S_n'(\beta) F_n(\alpha) - m F_n'(\alpha) S_n(\beta)}$$

$$b_n = (-1)^{n+\frac{3}{2}} (2n+1) \frac{m S_n(\alpha) S_n'(\beta) - S_n(\beta) S_n'(\alpha)}{m F_n(\alpha) S_n'(\beta) - S_n(\beta) F_n'(\alpha)}$$

and

$$S_n(x) = \left(\tfrac{\pi x}{2}\right)^{1/2} J_{n+\frac{1}{2}}(x)$$

$$F_n(x) = S_n(x) + iC_n(x)$$

$$C_n(x) = (-1)^n \left(\tfrac{\pi x}{2}\right)^{1/2} J_{-n-\frac{1}{2}}(x)$$

$$z_n = z_n(x) = \frac{1}{n(n+1)} \frac{\partial b_n(x)}{\partial x}$$

$$\alpha = \frac{2\pi r}{\lambda}$$

$$\beta = m\alpha$$

$$X = \cos\theta$$

The quantities $J_{n+\frac{1}{2}}(x)$ and $J_{-n-\frac{1}{2}}(x)$ are Bessel functions of

half integral order and $\dfrac{b_n(x)}{n(n+1)}$ is the Legendre polynomial of

degree n. Primes denote a first-order derivative.

The expressions for $i_1(\theta)$ and $i_2(\theta)$ are both infinite series. Each term in this series is made up of two parts - a magnetic and an electric component. The term α_n is the magnetic component and the term b_n is the electric component. For the case where $r \ll \lambda$, the terms of this series diminish rapidly and a satisfactory approximation is given by including only the electric component of the first term. The case of $r \ll \lambda$ is the case of a simple electric dipole. The results of using this approximation agree with the relation determined by Rayleigh for very small particles.

Scattering-Area-Coefficient Expression

The integral expression for the scattering-area coefficient k, as given in reference 6, is

$$k = \frac{1}{\alpha^2} \int_0^\pi \left[i_1(\theta) + i_2(\theta) \right] \sin\theta \, d\theta$$

where θ is the angle between the incident beam and the reverse direction of observed scattered light and $i_1(\theta)$ and $i_2(\theta)$ are defined in the first section of this appendix. The integral term on the right represents the total scattered light.

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pp. 337-340.

TABLE 1.- EXTENSION OF TABLES OF SCATTERING-AREA

COEFFICIENTS FOR AN INDEX OF

REFRACTION   m = 1.20

α	k	α	k
0.1	4.354×10^{-6}	0.5	2.588×10^{-3}
.2	6.925×10^{-5}	.6	5.233×10^{-3}
.3	3.471×10^{-4}	.8	1.543×10^{-2}
.4	1.081×10^{-3}	1.0	3.405×10^{-2}
		1.2	6.167×10^{-2}

TABLE 2.- EXTENSION OF TABLES OF ANGULAR-LIGHT-INTENSITY-DISTRIBUTION FUNCTIONS

FOR AN INDEX OF REFRACTION m = 1.20

| θ (deg)	$i_1(\theta)$	$i_2(\theta)$	$i_1(\theta)$	$i_2(\theta)$	$i_1(\theta)$	$i_2(\theta)$
	$\alpha = 0.1$		$\alpha = 0.2$		$\alpha = 0.3$	
20	1.626×10^{-8}	1.436×10^{-8}	1.022×10^{-6}	9.026×10^{-7}	1.130×10^{-5}	9.971×10^{-6}
30	1.627×10^{-8}	1.220×10^{-8}	1.024×10^{-6}	7.674×10^{-7}	1.133×10^{-5}	8.489×10^{-6}
40	1.628×10^{-8}	9.549×10^{-9}	1.025×10^{-6}	6.012×10^{-7}	1.137×10^{-5}	6.663×10^{-6}
140	1.638×10^{-8}	9.615×10^{-9}	1.052×10^{-6}	6.180×10^{-7}	1.206×10^{-5}	7.088×10^{-6}
150	1.639×10^{-8}	1.229×10^{-8}	1.054×10^{-6}	7.909×10^{-7}	1.210×10^{-5}	9.085×10^{-6}
160	1.639×10^{-8}	1.448×10^{-8}	1.055×10^{-6}	9.321×10^{-7}	1.214×10^{-5}	1.072×10^{-5}
	$\alpha = 0.4$		$\alpha = 0.5$		$\alpha = 0.6$	
20	6.078×10^{-5}	5.363×10^{-5}	2.190×10^{-4}	1.931×10^{-4}	6.083×10^{-4}	5.363×10^{-4}
30	6.109×10^{-5}	4.574×10^{-5}	2.207×10^{-4}	1.651×10^{-4}	6.154×10^{-4}	4.597×10^{-4}
40	6.151×10^{-5}	3.599×10^{-5}	2.231×10^{-4}	1.303×10^{-4}	6.251×10^{-4}	3.641×10^{-4}
140	6.825×10^{-5}	4.017×10^{-5}	2.625×10^{-4}	1.548×10^{-4}	7.903×10^{-4}	4.669×10^{-4}
150	6.871×10^{-5}	5.162×10^{-5}	2.653×10^{-4}	1.995×10^{-4}	8.022×10^{-4}	6.039×10^{-4}
160	6.905×10^{-5}	6.102×10^{-5}	2.673×10^{-4}	2.363×10^{-4}	8.111×10^{-4}	7.174×10^{-4}
	$\alpha = 0.8$		$\alpha = 1.0$		$\alpha = 1.2$	
20	2.806×10^{-3}	2.470×10^{-3}	8.053×10^{-3}	7.069×10^{-3}	1.605×10^{-2}	1.403×10^{-2}
30	2.866×10^{-3}	2.134×10^{-3}	8.337×10^{-3}	6.169×10^{-3}	1.695×10^{-2}	1.241×10^{-2}
40	2.949×10^{-3}	1.707×10^{-3}	8.736×10^{-3}	5.000×10^{-3}	1.823×10^{-2}	1.025×10^{-2}
140	4.495×10^{-3}	2.670×10^{-3}	1.714×10^{-2}	1.027×10^{-2}	5.026×10^{-2}	3.042×10^{-2}
150	4.616×10^{-3}	3.484×10^{-3}	1.787×10^{-2}	1.355×10^{-2}	5.339×10^{-2}	4.071×10^{-2}
160	4.707×10^{-3}	4.168×10^{-3}	1.842×10^{-2}	1.634×10^{-2}	5.581×10^{-2}	4.963×10^{-2}

NACA
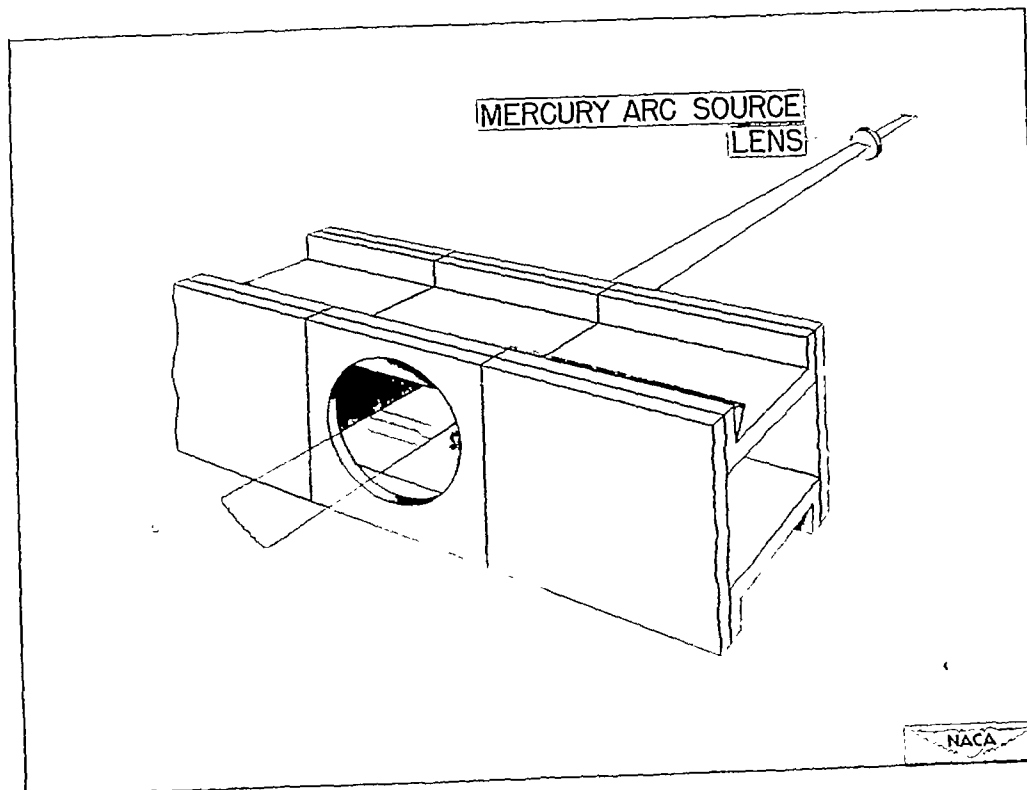
Figure 1.- Setup of light-scattering experiment. (From reference 1.)
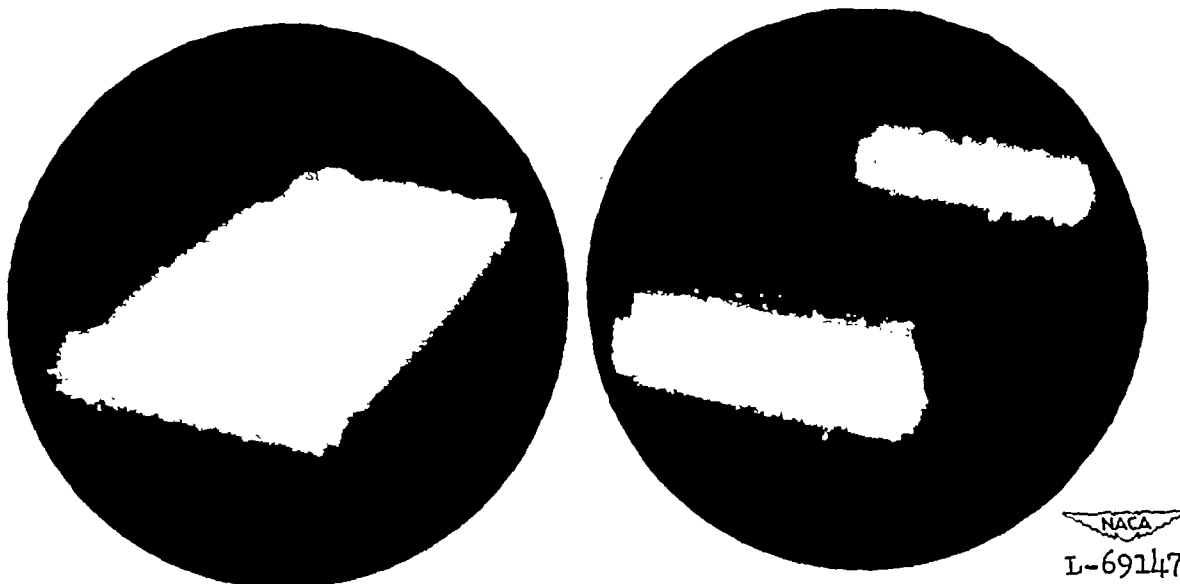
(a) No preheating of air. Stagnation temperature of 540° F absolute.

(b) Air preheated. Stagnation temperature of 1160° F absolute.

Figure 2.- Photograph of light beam passing through test section of Langley 11-inch hypersonic tunnel. (From reference 1.)
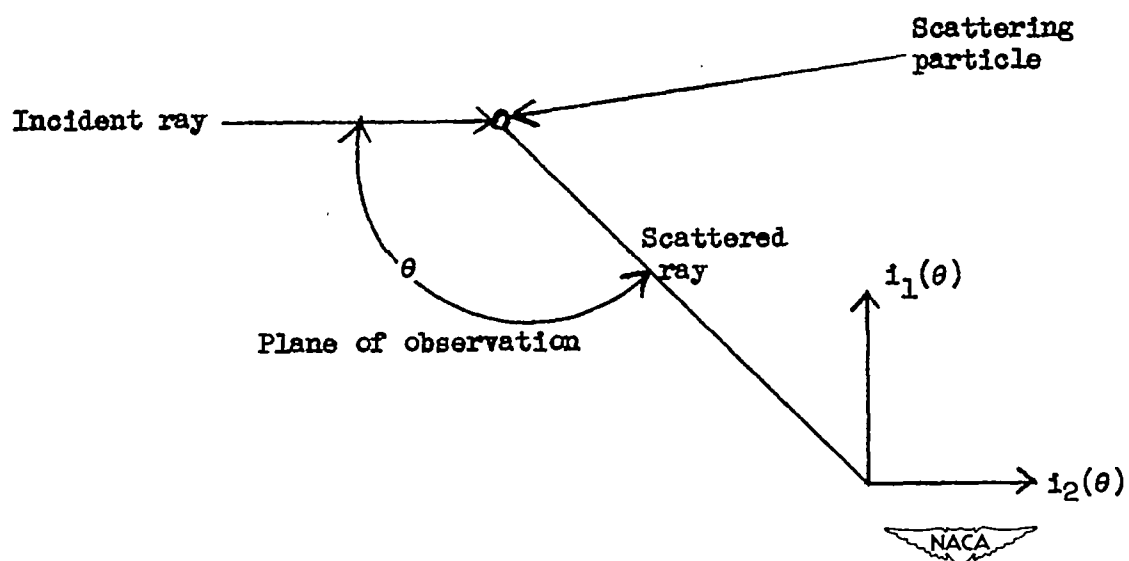
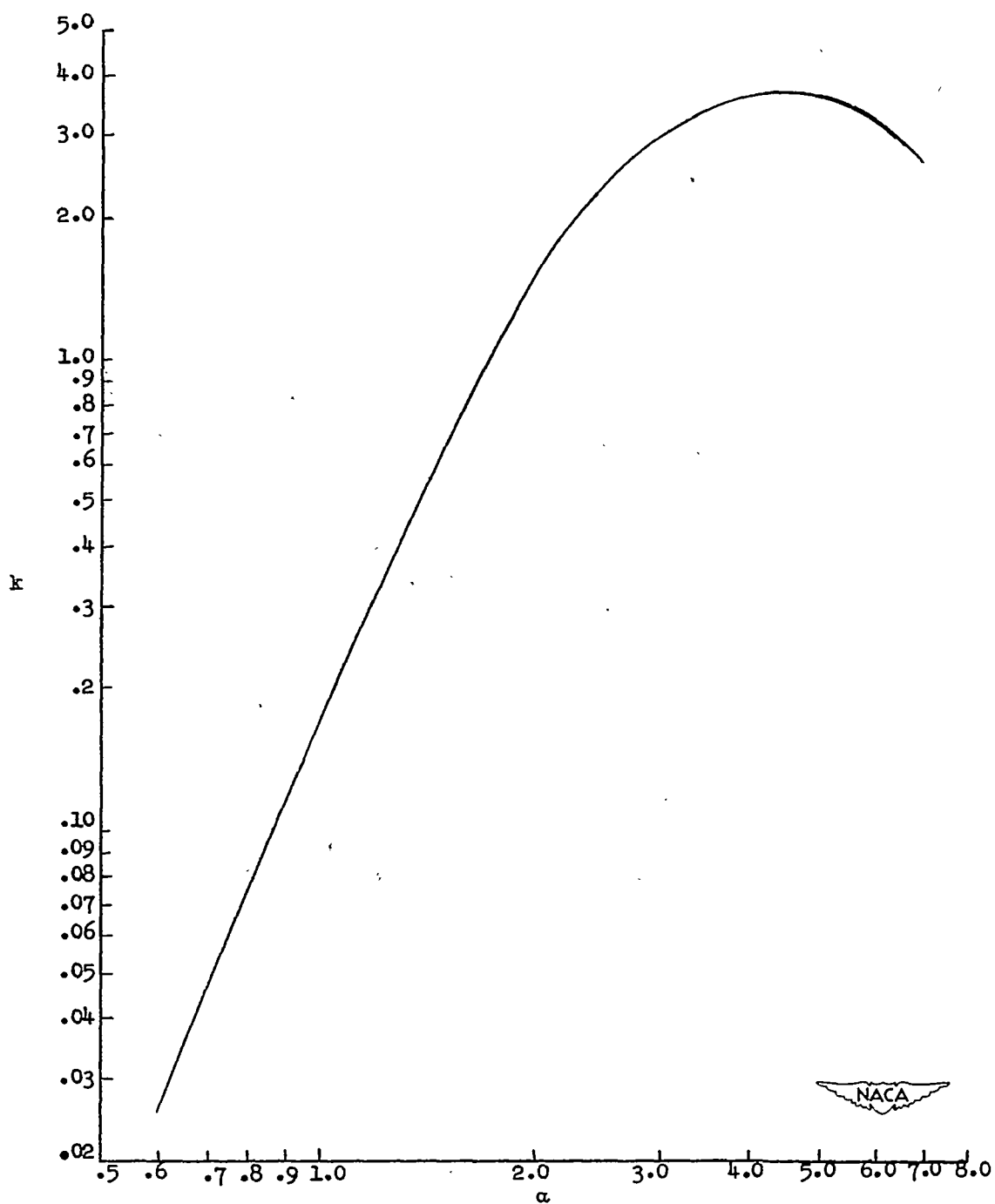Figure 3.- Directional notation used in light-scattering experiments.

Figure 4.- Illustrative curve of scattering-area coefficient k as a
function of α for index of refraction 1.44. (Data from reference 6.)

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A   High-voltage photocell power supply

B   RCA 1P21 multiplier phototube

C   Ballantine model 304 vacuum-tube voltmeter

D   Chamber for generating ammonium-chloride fog

E   Glass windows

F   Wratten light-filter monochromat 77A

G   Focusing lens

H   Water infrared filter

I   Adjustable slit

J   Air-cooled mercury-vapor high-
    pressure lamp GE type B-H6
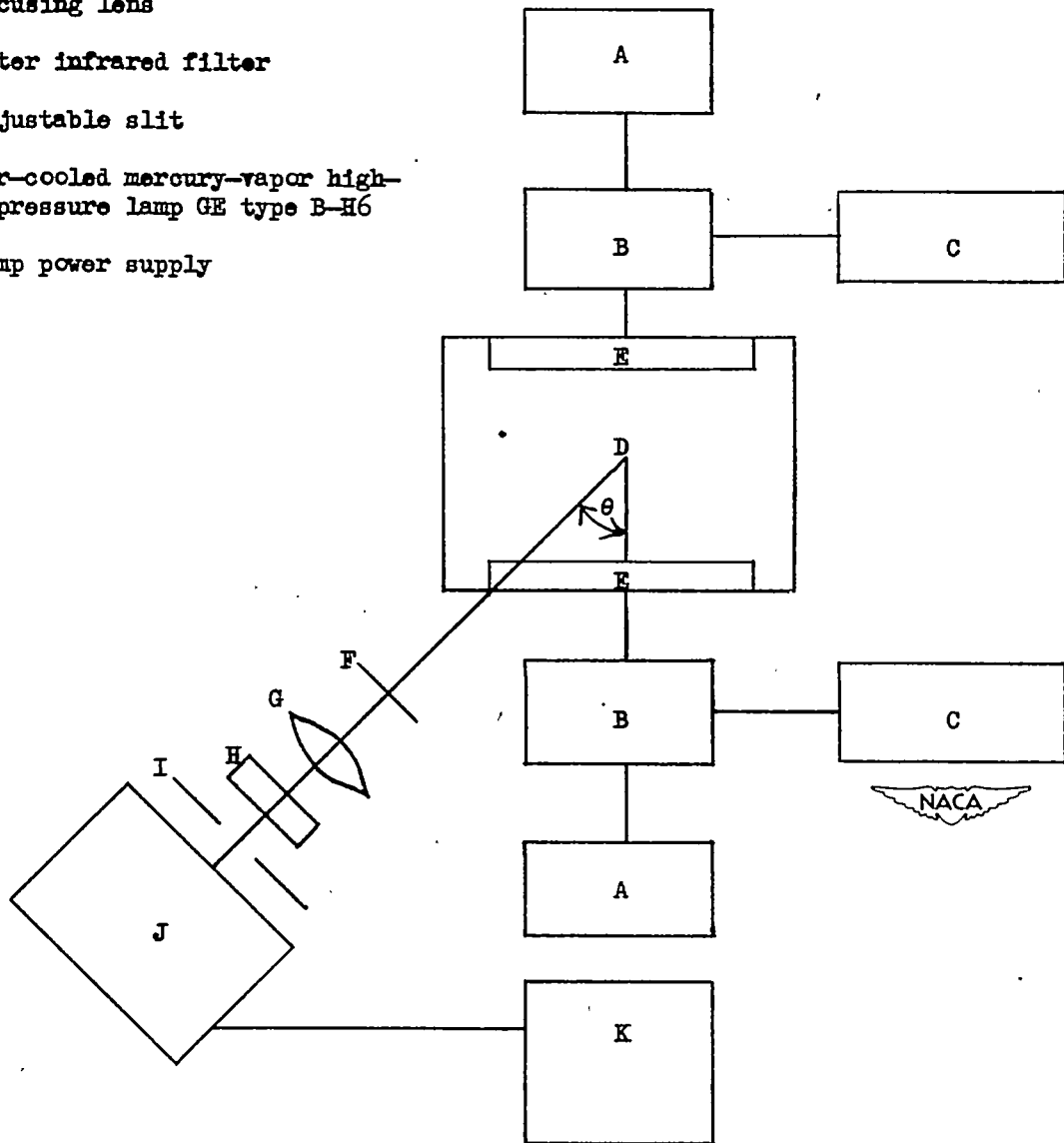
K   Lamp power supply
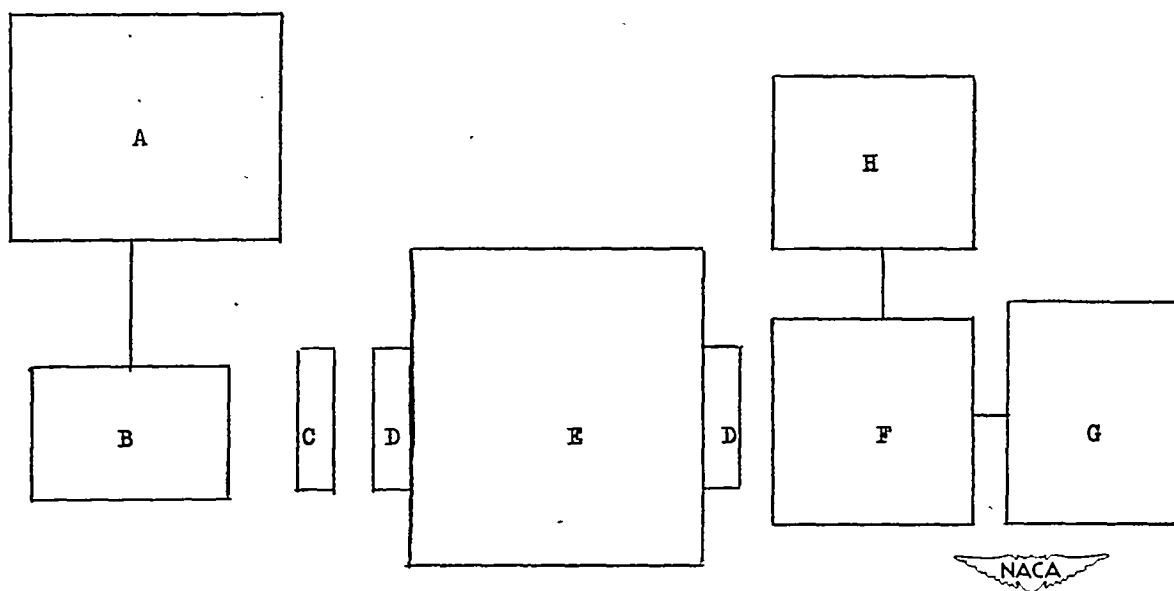

Figure 5.- Experimental arrangement used in measuring particle sizes in
           ammonium-chloride fog by light-scattering methods.

A Lamp power supply

B Low-pressure mercury lamp

C Wratten light-filter monochromat 77A

D Glass windows

E Chamber for generating ammonium-chloride fog

F RCA 1P21 multiplier phototube

G Ballantine model 304 vacuum-tube voltmeter

H Photocell power supply

Figure 6.- Experimental arrangement used in measuring particle size and
concentration by light-transmission method.

A   High-voltage photocell power supply

B   RCA 1P21 multiplier phototube

C   Narrow-band amplifier

D   Test section of the Langley 11-inch hypersonic tunnel

E   Glass windows

F   Wratten light-filter monochromat 77A

G   Focusing lens

H   Water infrared filter

I   Adjustable slit

J   Air-cooled mercury-vapor high-pressure lamp GE type B-H6

K   Lamp power supply
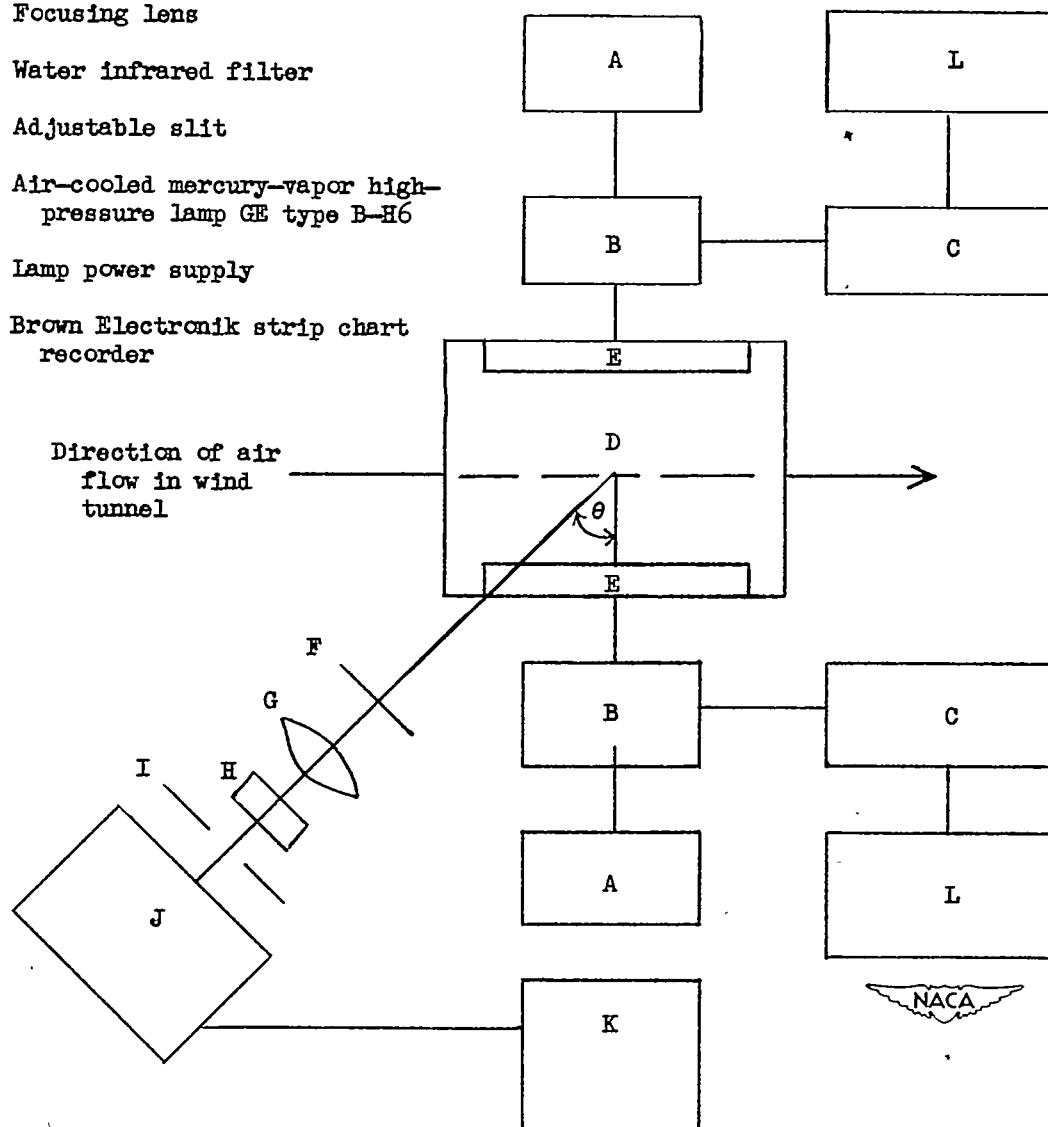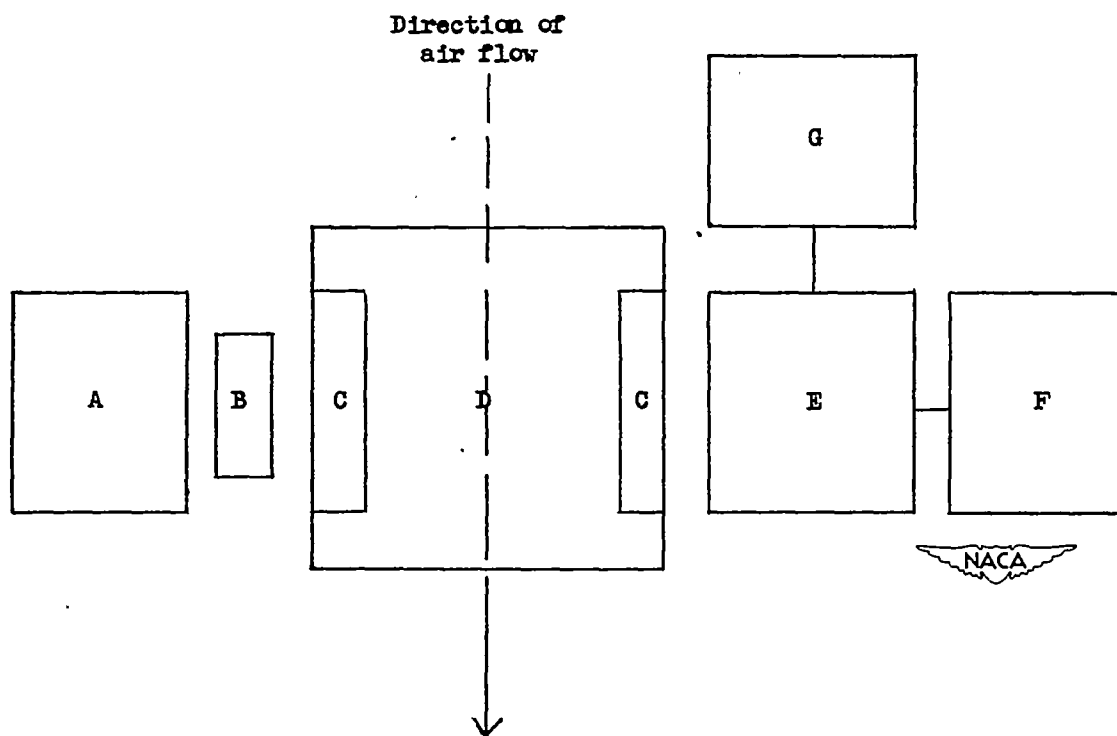
L   Brown Electronik strip chart recorder

Figure 7.- Experimental arrangement used in measuring particle sizes by light-scattering methods in condensation experiments in the Langley 11-inch hypersonic tunnel.

A   Current-stabilized pilot lamp

B   Wratten light-filter monochromat 77A

C   Glass windows

D   Test section of the Langley 11-inch hypersonic tunnel

E   RCA 1P21 multiplier phototube

F   Brown Electronik strip chart recorder

G   Photocell power supply

Figure 8.- Experimental arrangement used in measuring condensation particle
     concentration by light-transmission method in the Langley 11-inch
     hypersonic tunnel.